Financial Markets Microstructure
Final Re-Exam with Solutions

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Problem 1

This problem explores the Glosten-Milgrom model with feedback, in which the firm can use the stock market to gauge the attractiveness of an investment project.

In particular, suppose that the firm is facing a binary investment decision. If it invests in the project (e.g., decides to develop a new product), this project will yield net return v_ω, which depends on the state of the world $\omega \in \{l, h\}$ with $\mathbb{P}(h) = 1/2$. If the firm does not invest, it gets zero. The returns are such that $v_h > 0 > v_l$, i.e., the firm wants to invest in the project if and only if the state is $\omega = h$. The baseline value of the firm is μ; it changes to $\mu + v_\omega$ if the firm invests and remains at μ otherwise.

The timeline is as follows: the firm announces the investment project to the public, one period of trading in the financial market follows, the firm observes trading outcomes and decides whether to proceed with investing in the project or not.

The financial market is modelled as a standard Glosten-Milgrom setting: one trader can submit a buy or a sell order for one unit of the asset. The trader is a profit-maximizing insider with probability $\pi \in (0, 1)$, in which case he knows the true state ω. (Think of the insider as an expert in this industry.) With probability $1 - \pi$, the trader is a noise trader, who submits a buy or a sell order with equal probabilities regardless of ω. Orders are executed by a representative competitive dealer, who provides bid and ask quotes.

1. Suppose that the insider buys the asset when $\omega = h$ and sells when $\omega = l$.

   (a) What is the expected net value of investment for the firm when it observes a buy order in the market? When it observes a sell order?

   (b) For which values of π is it optimal for the firm to "follow the market", i.e., to proceed with the investment when its announcement generates demand for its stocks and to revert its decision when the announcement triggers a "sell-off"? What does this condition mean intuitively?

(c) Assuming that the condition you derived in (b) holds and that the firm thus follows the market, derive the bid and ask prices quoted by the dealer.

(d) Assuming the condition from (b) holds and given everything you derived, is it optimal for the insider to follow the strategy we assumed? Conclude whether the situation described above constitutes an equilibrium.

2. Assume now the condition from (1b) does not hold and that $\bar{v} = \frac{v_h+v_l}{2} < 0$. Derive formally the pure-strategy equilibrium that occurs in this case. Explain intuitively what happens in this equilibrium and why.

3. Assume now the condition from (1b) does not hold and that $\bar{v} = \frac{v_h+v_l}{2} > 0$. Derive formally the pure-strategy equilibrium that occurs in this case. Explain intuitively what happens in this equilibrium and why.

Solution:

Part 1.

(a) The probability that all initially uninformed parties (the firm and the dealer) assign to the state being $\omega = h$ after observing a Buy order is

$$\mathbb{P}(h|\text{Buy}) = \frac{\frac{\pi}{2} + \frac{1-\pi}{4}}{\frac{\pi}{2} + \frac{1-\pi}{2}} = \frac{1+\pi}{2}.$$

The expected return on investment after observing a Buy order is then

$$\mathbb{E}(v_\omega|\text{Buy}) = \frac{1+\pi}{2}v_h + \left(1 - \frac{1+\pi}{2}\right)v_l = \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l.$$

For the sale order the two are equal to, respectively:

$$\mathbb{P}(h|\text{Sell}) = \frac{\frac{1-\pi}{4}}{\frac{\pi}{2} + \frac{1-\pi}{2}} = \frac{1-\pi}{2},$$
$$\mathbb{E}(v_\omega|\text{Sell}) = \frac{1-\pi}{2}v_h + \left(1 - \frac{1-\pi}{2}\right)v_l = \frac{1-\pi}{2}v_h + \frac{1+\pi}{2}v_l.$$

(b) For it to be optimal for the firm to follow the market's advice, it must be that $\mathbb{E}(v_\omega|\text{Buy}) \geq 0 \geq \mathbb{E}(v_\omega|\text{Sell})$. The former inequality is equivalent to

$$\frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l \geq 0$$
$$\iff \pi \geq \frac{-2\bar{v}}{v_h - v_l},$$

where $\bar{v} = \frac{v_h+v_l}{2}$. If $\bar{v} \geq 0$ then this condition always holds. Similarly, $\mathbb{E}(v_\omega|\text{Sell}) \leq 0$ is equivalent to $\pi \geq \frac{2\bar{v}}{v_h-v_l}$. This always holds if $\bar{v} \leq 0$. Both conditions are satisfied if

$$\pi \geq \frac{2|\bar{v}|}{v_h - v_l}. \tag{$\star$}$$

Intuitively, this condition requires that there are enough insiders in the market for the price to be informative. If $\bar{v} = 0$ then the firm is ex ante indifferent between investing and not, so any positive or negative signal could convince it to do the respective decision. But if, for example, $\bar{v} > 0$, then the investment project looks appealing ex ante, so the negative price signal should be informative enough of ω for the firm to decide to go back on its investment decision.

(c) Let $I \in \{0, 1\}$ denote the firm's final investment decision. Then $I = 1$ after a buy order and $I = 0$ after a sell order. The dealer is competitive, so we can obtain the quoted prices from the zero-profit condition:

$$a = \mathbb{E}(\mu + Iv_\omega \mid \text{Buy}) = \mu + \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l; \tag{1}$$

$$b = \mathbb{E}(\mu + Iv_\omega \mid \text{Sell}) = \mu. \tag{2}$$

(d) If $\omega = h$, the insider's profit is

$$\Pi_s(h) = \begin{cases} (\mu + v_h) - \left(\mu + \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l\right) = \frac{1-\pi}{2}(v_h - v_l) > 0 & \text{if Buy;} \\ 0 & \text{if Pass;} \\ (\mu) - (\mu) = 0 & \text{if Sell} \end{cases}$$

Buying is thus the optimal decision in state $\omega = h$. On the other hand, if $\omega = l$:

$$\Pi_s(l) = \begin{cases} (\mu + v_l) - \left(\mu + \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l\right) = -\frac{1+\pi}{2}(v_h - v_l) < 0 & \text{if Buy;} \\ 0 & \text{if Pass;} \\ (\mu) - (\mu) = 0 & \text{if Sell} \end{cases}$$

so selling is weakly optimal.

In the end, if ($\star$) holds, we have an equilibrium, in which the dealer quotes the ask and bid prices as given by (1) and (2) respectively, the informed trader buys if $\omega = h$ and sells if $\omega = l$, and the firm invests if and only if it observes a buy order (equivalently, if the price of its stock goes up after the investment project is announced).

Part 2. Now ($\star$) is violated and $\bar{v} < 0$, meaning that the firm never invests in the project, regardless of how the market reacts. This means that $\mathbb{E}(\mu + Iv_\omega \mid \text{Buy}) = \mathbb{E}(\mu + Iv_\omega \mid \text{Sell}) = \mu$ because $I = 0$. The equilibrium then is such that the dealer sets $a = b = \mu$, the firm never invests, and the insider's strategy is arbitrary (since all actions yield zero profit).

Since the firm never invests, the insider's knowledge is irrelevant to the firm value, and there is no adverse selection in the market.

Part 3. If ($\star$) is violated and $\bar{v} > 0$, the firm always invests. In that case, it is weakly optimal for the insider to buy if $\omega = h$, since then $\Pi_s(h) = \mu + v_h - a$ and $a = \mathbb{E}(\mu + v_\omega \mid \text{Buy}) \leq \mu + v_h$, and by selling the insider would get $\Pi_s(h) = b - (\mu + v_h) \leq 0$ because $b \leq \mu + v_h$. Similarly, it is weakly optimal for the insider to sell if $\omega = l$. Assuming the insider behaves this way, we can then derive the dealer's quotes as

$$a = \mathbb{E}(\mu + Iv_\omega \mid \text{Buy}) = \mu + \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l;$$
$$b = \mathbb{E}(\mu + Iv_\omega \mid \text{Sell}) = \mu + \frac{1-\pi}{2}v_h + \frac{1+\pi}{2}v_l.$$

Therefore, there exists an equilibrium, in which the dealer sets the prices as above, the insider buys if $\omega = h$ and sells if $\omega = l$, and the firm always invests in the project.

In this case the project is so appealing ex ante that the market can not dissuade the firm from pursuing it. The insider's trading thus does not affect the firm's investment decisions, and we are back to the standard Glosten-Milgrom model.

As in part 1 above, there may, in principle, exist mixed-strategy equilibria in this case, in which the speculator is indifferent in one of the states and mixes between buying and passing or between selling and passing.

Problem 2

We have discussed in class that corporate bond markets operate on the basis of RFQs (requests for quotes). The reality is slightly more intricate. The traders typically have a choice between calling a dealer on the phone (voice trading) and using an electronic platform to submit RFQs to a set of dealers (electronic trading). These two methods of trading have coexisted for some time, with electronic trading gradually gaining market share.

Answer the following questions to the best of your ability, relying on the knowledge you have obtained throughout the course. Provide at least two reasons/arguments/suggestions when answering each question.

1. Given the option to trade electronically, why could the traders and dealers prefer to use voice

trading?

2. There is some evidence that the spread of electronic trading has led to better quotes being offered in voice trading, and that dealers with more electronic trading in a given bond tend to provide better prices in their voice trading. Why, in your opinion, could this happen?

3. Suppose you are contracted as a consultant by a small electronic exchange, with the goal of increasing the market share of this exchange in corporate bond trading. What suggestions can you give to the exchange that would allow it to attract trading flow?

Solution: This problem follows many points and observations mentioned by O'Hara and Zhou (2021).[1]

Part 1.

(a) This may be the result of a miscoordination (as we discussed when talking about market fragmentation): traders do not engage in electronic markets because dealers are not present there, and dealers do not enter the electronic markets because there are no traders there.

(b) Advertising a trade among many dealers on the electronic market may not be desirable to a trader if this trade is based on private information. Advertising the trade would thus reveal the trader's private information and limit the potential profits the trader could extract from this information. Voice trading, on the other hand, limits the disclosure of information to the particular dealer the trader is engaging with, and is thus more beneficial for such trades. As we discussed in relation to market transparency, the dealers also prefer having informational advantage relative to the rest of the market, and so they would be willing to offer low commissions on voice trading in order to acquire this private information about order flow.

Part 2.

(a) Increasing competition from electronic trading venues can force dealers to provide more competitive prices in their voice trading (related to our discussion of trading costs in fragmented markets).

(b) Electronic trading reduces the costs for searching for the right counterparties. So dealers who are more active in the electronic market are able to unravel their inventory more easily, and due to this they are able to provide better quotes to their clients in voice trading.

(c) Dealers' pricing in their traditional voice trading could be improved by information they learn from both trade interests and actual trades on electronic trading platforms. Having more information about asset value means that dealers are less exposed to adverse selection and are able to quote tighter spreads.

Part 3. This question builds on top of our discussion of market fragmentation and market transparency; any of the many arguments used in those discussions could be invoked in this answer. Below are some examples.

(a) The first suggestion is to be an appealing platform: offer low trading costs, convenient interface, and be convenient to use in all other respects.

(b) As we discussed, liquidity begets liquidity, meaning that offering good service is not by itself sufficient to attract business – the platform needs to attract a critical mass of trades before it can attract more. One option is to negotiate with dealers from other platforms and to attempt to lure them into the platform (with monetary incentives like rebates). Another option is to employ new market makers who would provide liquidity in the market.

(c) In addition to attracting traders with better liquidity and lower trading costs, the platform can make transparency guarantees, making it easy for traders to access information about quotes, past trades, and counterparty identity. As we argued, this would attract uninformed traders to the market (which, in turn, would allow dealers to supply better quotes and further improve liquidity).

[1] O'Hara, Maureen, and Xing Alex Zhou. "The Electronic Evolution of Corporate Bond Dealers." Journal of Financial Economics 140, no. 2 (May 1, 2021): 368–90. https://doi.org/10.1016/j.jfineco.2021.01.001.